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Palatini formulation of modified gravity with $\ln R$ terms

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Abstract

Recently, corrections to the standard Einstein–Hilbert action are proposed to explain the current cosmic acceleration instead of introducing dark energy. We discuss the Palatini formulation of the modified gravity with a $\ln R$ term suggested by Nojiri and Odintsov. We show that in the Palatini formulation, the $\ln R$ gravity can drive a current exponential accelerated expansion and it reduces to the standard Friedmann evolution for high redshift region. We also discuss the equivalent scalar–tensor formulation of the theory. We indicate that the $\ln R$ gravity may still have a conflict with electron–electron scattering experiment which stimulates us to pursue a more fundamental theory which can give the $\ln R$ gravity as an effective theory. Finally, we discuss a problem faced with the extension of the $\ln R$ gravity by adding R^m terms.

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1. Introduction

That our universe expansion is currently in an accelerating phase now seems well-established. The most direct evidence for this is from the measurement of type Ia supernova [1]. Other indirect evidences such as the observations of CMB by the WMAP satellite [2], large-scale galaxy surveys by 2dF and SDSS also seem supporting this.

But now the mechanism responsible for this acceleration is not very clear. Many authors introduce a mysterious cosmic fluid called dark energy to explain this (see Refs. [3–5] for a review). On the other hand, some authors suggest that maybe there does not exist

such mysterious dark energy, but the observed cosmic acceleration is a signal of our first real lack of understanding of gravitational physics [6]. An example is the braneworld theory of Dvali et al. [7].

Recently, some authors proposed to add a R^{-1} term in the Einstein–Hilbert action to modify the general relativity (GR) [8,9]. It is interesting that such terms may be predicted by string/M-theory [10]. It was shown in their work that this additional term can give accelerating solutions of the field equations without dark energy. Based on this modified action, Vollick [11] used Palatini variational principle to derive the field equations. In the Palatini formalism, instead of varying the action only with respect to the metric, one views the metric and connection as independent field variables and vary the action with respect to them independently. This would give second order field equations. In the original Einstein–

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Hilbert action, this approach gives the same field equations as the metric variation. For a more general action, those two formalism are inequivalent, they will lead to different field equations and thus describe different physics [12]. Flanagan [13] derived the equivalent scalar–tensor theory of the Palatini formulation. Furthermore, in Ref. [14], Flanagan derived the equivalent scalar–tensor theory of a more general modified gravity framework. Those results are very important and fundamental for the Palatini formalism. We will apply his framework in Section 3 to discuss the $\ln R$ gravity. In Ref. [15], Dolgov and Kawasaki argued that the fourth order field equations following from the metric variation suffer serious instability problem. If this is indeed the case, the Palatini approach appears even more appealing, because the second order field equations following from Palatini variation are free of this sort of instability [16]. However, the most convincing motivation to take the Palatini formalism seriously is that the field equations following from it fit the SN Ia data at an acceptable level [16]. An extension of the $1/R$ theory, the $R + 1/R + R^2$ theory has been discussed in metric formation by Nojiri and Odintsov [17]. It is shown that such an extension may explain both the current acceleration and early inflation and it may resolve the instability of the original $1/R$ gravity. Its Palatini formation is discussed in Ref. [18]. Interestingly, in the Palatini formation, while it can still drive a current acceleration, adding a R^2 term cannot drive a early inflation. The difference of metric formation and Palatini formation is thus quite obvious. But now we still cannot tell which one is physical.

In Ref. [19], Nojiri and Odintsov presented another effort in this direction to modify gravity theory. They added a $\ln R$ term to the Einstein–Hilbert action. They considered the metric formation of this theory and concluded that such a theory can derive an accelerated expansion. By the above considerations, we think it is worth further investigating of the Palatini formulation of this $\ln R$ theory.

This Letter is arranged as follows: in Section 2 we derive the modified Friedmann (MF) equation in Palatini formulation of the $\ln R$ theory and discuss several of its features; in Section 3 we discuss the equivalent scalar–tensor formulation of the $\ln R$ theory and an extension of the $\ln R$ theory also suggested

by Nojiri and Odintsov [19]; Section 4 is devoted to conclusions and discussions.

2. The model and the modified Friedmann equation

Firstly, we briefly review deriving field equations from a generalized Einstein–Hilbert action by using Palatini variational principle. See Refs. [11,12,16,18] for details. We will follow the sign conventions of Ref. [20] in this Letter.

The field equations follow from the variation in Palatini approach of the generalized Einstein–Hilbert action

$$S = -\frac{1}{2\kappa} \int d^4x \sqrt{-g} L(R) + \int d^4x \sqrt{-g} L_M, \quad (1)$$

where $\kappa = 8\pi G$, L is a function of the scalar curvature R and L_M is the Lagrangian density for matter.

Varying with respect to $g_{\mu\nu}$ gives

$$L'(R)R_{\mu\nu} - \frac{1}{2}L(R)g_{\mu\nu} = -\kappa T_{\mu\nu}, \quad (2)$$

where a prime denotes differentiation with respect to R and $T_{\mu\nu}$ is the energy–momentum tensor given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}. \quad (3)$$

We assume the universe contains dust and radiation, denoting their energy densities as ρ_m and ρ_r , respectively, thus $T_{\nu}^{\mu} = \{-\rho_m - \rho_r, p_r, p_r, p_r\}$ and $T = g^{\mu\nu}T_{\mu\nu} = -\rho_m$ because of the relation $p_r = \rho_r/3$.

In the Palatini formulation, the connection is not associated with $g_{\mu\nu}$, but with $h_{\mu\nu} \equiv L'(R)g_{\mu\nu}$, which is known from varying the action with respect to $\Gamma_{\mu\nu}^{\lambda}$. Thus the Christoffel symbol with respect to $h_{\mu\nu}$ is given by

$$\Gamma_{\mu\nu}^{\lambda} = \{\lambda_{\mu\nu}\}_g + \frac{1}{2L'} [2\delta_{(\mu}^{\lambda} \partial_{\nu)} L' - g_{\mu\nu} g^{\lambda\sigma} \partial_{\sigma} L'], \quad (4)$$

where the subscript g signifies that this is the Christoffel symbol with respect to the metric $g_{\mu\nu}$.

The Ricci curvature tensor is given by

$$R_{\mu\nu} = R_{\mu\nu}(g) - \frac{3}{2}(L')^{-2} \nabla_{\mu} L' \nabla_{\nu} L' + (L')^{-1} \nabla_{\mu} \nabla_{\nu} L' + \frac{1}{2}(L')^{-1} g_{\mu\nu} \nabla_{\sigma} \nabla^{\sigma} L' \quad (5)$$

and

$$R = R(g) + 3(L')^{-1} \nabla_\mu \nabla^\mu L' - \frac{3}{2} (L')^{-2} \nabla_\mu L' \nabla^\mu L', \quad (6)$$

where $R_{\mu\nu}(g)$ is the Ricci tensor with respect to $g_{\mu\nu}$ and $R = g^{\mu\nu} R_{\mu\nu}$. Note by contracting (2), we get

$$L'(R)R - 2L(R) = -\kappa T. \quad (7)$$

Assume we can solve R as a function of T from (7). Thus (5), (6) do define the Ricci tensor with respect to $h_{\mu\nu}$.

Then we review the general framework of deriving modified Friedmann equation in Palatini formulation [16]. Consider the Robertson–Walker metric describing the cosmological evolution,

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2). \quad (8)$$

We only consider a flat metric, which is favored by present observations [2].

From Eqs. (8) and (5), we can get the non-vanishing components of the Ricci tensor:

$$R_{00} = 3\frac{\ddot{a}}{a} - \frac{3}{2}(L')^{-2}(\partial_0 L')^2 + \frac{3}{2}(L')^{-1} \nabla_0 \nabla_0 L', \quad (9)$$

$$R_{ij} = -\left[a\ddot{a} + 2\dot{a}^2 + (L')^{-1} \Gamma_{ij}^0 \partial_0 L' + \frac{a^2}{2} (L')^{-1} \nabla_0 \nabla_0 L' \right] \delta_{ij}. \quad (10)$$

Substituting Eqs. (9) and (10) into the field equations (2), we can get

$$6H^2 + 3H(L')^{-1} \partial_0 L' + \frac{3}{2} (L')^{-2} (\partial_0 L')^2 = \frac{\kappa(\rho + 3p) - L}{L'}, \quad (11)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, ρ and p is the total energy density and total pressure, respectively. Assume we can solve R in term of T from Eq. (7), substituting it to the expression for L' and $\partial_0 L'$, we can get the MF equation.

Now we turn to the consideration of the following modified Einstein–Hilbert action suggested by Nojiri and Odintsov [19]

$$L(R) = R - \beta \ln \frac{R}{-\alpha}. \quad (12)$$

Since our interest is to explain cosmic acceleration, we will assume $R < 0$ in this Letter, i.e., de Sitter space. Thus $\alpha > 0$.

The contracted field equation (7) now reads

$$f(R) \equiv \frac{R}{-\beta} + 2 \ln \frac{R}{-\alpha} - 1 = -\kappa T / \beta = \frac{\kappa \rho_m}{\beta}. \quad (13)$$

If $\beta > 0$, $f(R)$ is a monotonically decreasing function and we have $\lim_{R \rightarrow 0} f(R) \rightarrow -\infty$ and $\lim_{R \rightarrow -\infty} f(R) \rightarrow +\infty$. Thus R is uniquely determined for any value of $\kappa \rho_m / \beta \equiv x$ through Eq. (13). Let us denote it simply as $R = R(\kappa \rho_m / \beta) = R(x)$. Note that irrespective of the precise form of the relation $R(x)$, this is just an algebraic relation. Thus for a given T , there is no instabilities present in the metric formulation of the $1/R$ theory indicated by Dolgov and Kawasaki [15], whose origin is due to the fact that R is determined by a differential equation for a given T . To simply discussion, we will assume $\beta > 0$ from now on. Note that when $\alpha = \beta$, the vacuum solution $R_0 \equiv R(0)$ can be solved exactly as $R_0 = -\alpha$.

From the conservation equation $\dot{\rho}_m + 3H\rho_m = 0$ we can get

$$\partial_0 L' = \frac{3}{(R(x)/\beta)^2 - 2R(x)/\beta} \left(\frac{\kappa \rho_m}{\beta} \right) H \equiv F(x)H. \quad (14)$$

Substituting this to Eq. (11) we can get the Modified Friedmann equation:

$$H^2 = \frac{\kappa \rho_m + 2\kappa \rho_r - \beta \left(\frac{R}{\beta} - \ln \frac{R}{-\alpha} \right)}{(1 - \frac{\beta}{R})(6 + 3F(x)(1 + \frac{1}{2}F(x)))}. \quad (15)$$

It can be seen from Eqs. (13), (14) and (15) that when $\beta \rightarrow 0$, the MF equation will reduce continuously to the standard Friedmann equation. Thus, the $\ln R$ modification is a smooth and continuous modification.

Let us first discuss the cosmological evolution without matter and radiation. Define the parameter n as $R_0 = -\alpha e^{-n}$. Substitute this to the vacuum field equation $f(R) = 0$, we can get $\alpha = e''(2n+1)\beta$ and $R_0 = -(2n+1)\beta$. Substitute those to the vacuum MF equation and set $t = 0$, we have

$$H_0^2 = \frac{\beta(n+1)}{6(1 + \frac{1}{2n+1})}. \quad (16)$$

Thus when $\beta \sim H_0^2 \sim (10^{-33} \text{ eV})^2$ and $n > -1/2$, the $\ln R$ modified gravity can indeed drive a current expo-

nential acceleration compatible with the observation. The role of the parameter β is similar to a cosmological constant or the coefficient of the $1/R$ term in the $1/R$ gravity [16].

When the energy density of dust cannot be neglected, i.e., $\kappa\rho_m/\beta \gg 1$, $F(x) \sim 0$ and if α satisfies $|\ln(\kappa\rho_m/\alpha)| \ll \kappa\rho_m/\beta$, i.e., $\exp(-\kappa\rho_m/\beta) \ll \alpha/\beta \ll \exp(\kappa\rho_m/\beta)$, from Eq. (13), $R \sim -\kappa\rho_m$. Then the MF equation (15) reduces to the standard Friedmann equation

$$H^2 = \frac{\kappa}{3}(\rho_m + \rho_r). \quad (17)$$

Thus if $\exp(-\kappa\rho_{m,\text{BBN}}/\beta) \ll \alpha/\beta \ll \exp(\kappa\rho_{m,\text{BBN}}/\beta)$, where $\rho_{m,\text{BBN}}$ is the energy density of dust in the epoch of BBN, the $\ln R$ gravity can be consistent with the BBN constraints on the form of Friedmann equation [21]. One possible choice is $\alpha = \beta$, for which the vacuum solution can be solved exactly $R_0 = -\alpha$. Since $\beta \sim H_0^2$, the condition $\kappa\rho_m/\beta \gg 1$ breaks down only in recent cosmological time. Thus the universe evolves in the standard way until recently, when $\ln R$ term begins to dominate and drives the observed cosmic acceleration.

3. Scalar-tensor formulation of the model

Recently, Flanagan [13] derived the equivalent scalar-tensor theory of the Palatini form of modified gravity theory. We adopt his formalism and apply it to the $\ln R$ theory.

Following Flanagan, the $\ln R$ theory is equivalent to the theory:

$$\begin{aligned} \tilde{S}[\tilde{g}_{\mu\nu}, \Phi, \psi_m] \\ = \int d^4x \sqrt{-\tilde{g}} \left[-\frac{\tilde{R}}{2\kappa} - V(\Phi) \right] \\ + S_m \left[\exp\left(-\sqrt{\frac{2\kappa}{3}}\Phi\right) \tilde{g}_{\mu\nu}, \psi_m \right], \end{aligned} \quad (18)$$

where $\tilde{g}_{\mu\nu} = \exp(\sqrt{\frac{2\kappa}{3}}\Phi)g_{\mu\nu}$ is the metric in Einstein-frame [22], \tilde{R} is the scalar curvature associated with $\tilde{g}_{\mu\nu}$, ψ_m is the matter field and Φ is a fictitious scalar field that can be deleted from the field equations.

The potential V can be obtained by the standard procedure [13,17]

$$\begin{aligned} V(\Phi) = \frac{\beta}{2\kappa} \left[-1 + \ln \frac{\beta}{\alpha} - \ln \left(\exp\left(\sqrt{\frac{2\kappa}{3}}\Phi\right) - 1 \right) \right] \\ \times \exp\left(-2\sqrt{\frac{2\kappa}{3}}\Phi\right). \end{aligned} \quad (19)$$

See Fig. 1 for the case of $\alpha = \beta$. Since the α appears in the expression of V only as the constant term $\ln(\beta/\alpha)$, other cases would not differ from it essentially.

The field equations are

$$\tilde{G}_{\mu\nu} = -\kappa \left[V(\Phi) \tilde{g}_{\mu\nu} + \exp\left(\sqrt{\frac{2\kappa}{3}}\Phi\right) T_{\mu\nu} \right] \quad (20)$$

and

$$\begin{aligned} V'(\Phi) &= -\sqrt{\frac{\kappa}{6}} \exp\left(-2\sqrt{\frac{2\kappa}{3}}\Phi\right) T \\ &= \sqrt{\frac{\kappa}{6}} \exp\left(-2\sqrt{\frac{2\kappa}{3}}\Phi\right) \rho_m, \end{aligned} \quad (21)$$

where $T_{\mu\nu}$ is the Jordan-frame energy-momentum tensor defined by Eq. (3) and $T = g^{\mu\nu}T_{\mu\nu}$.

We can read off the evolution of Φ from Eq. (21) (see Fig. 2). In early universe, when ρ_m is large, Φ locates at large value; then as the universe evolves, while ρ_m dilutes to smaller and smaller value, correspondingly, Φ rolls down to the absolute minimum point of the potential at roughly $\sqrt{2\kappa/3}\Phi \sim 0.7$, at which it can drive an exponential acceleration expansion.

From Fig. 1, we can see that the energy scale of the absolute minimum of V is of order β/κ and as shown in Section 2, $\beta \sim (10^{-33} \text{ eV})^2$. Thus if we assume that the $\ln R$ theory is applicable in small scales such as the electron-electron scattering scale, there will be a severe conflict with particle experiment as shown explicitly by Flanagan [13] for the $1/R$ gravity. However, those modified gravity theory cannot be fundamental. They are effective theories. If it can be shown that their cut-off scale is much larger than the electron-electron scattering scale, the conflict will be fixed. This stimulates us to pursue their origin from more fundamental theory (see Ref. [10] for such an effort for the $1/R$ gravity). A large cut-off scale (or a small cut-off energy) is possible for modified gravity, e.g., for the effective field theory of massive gravity,

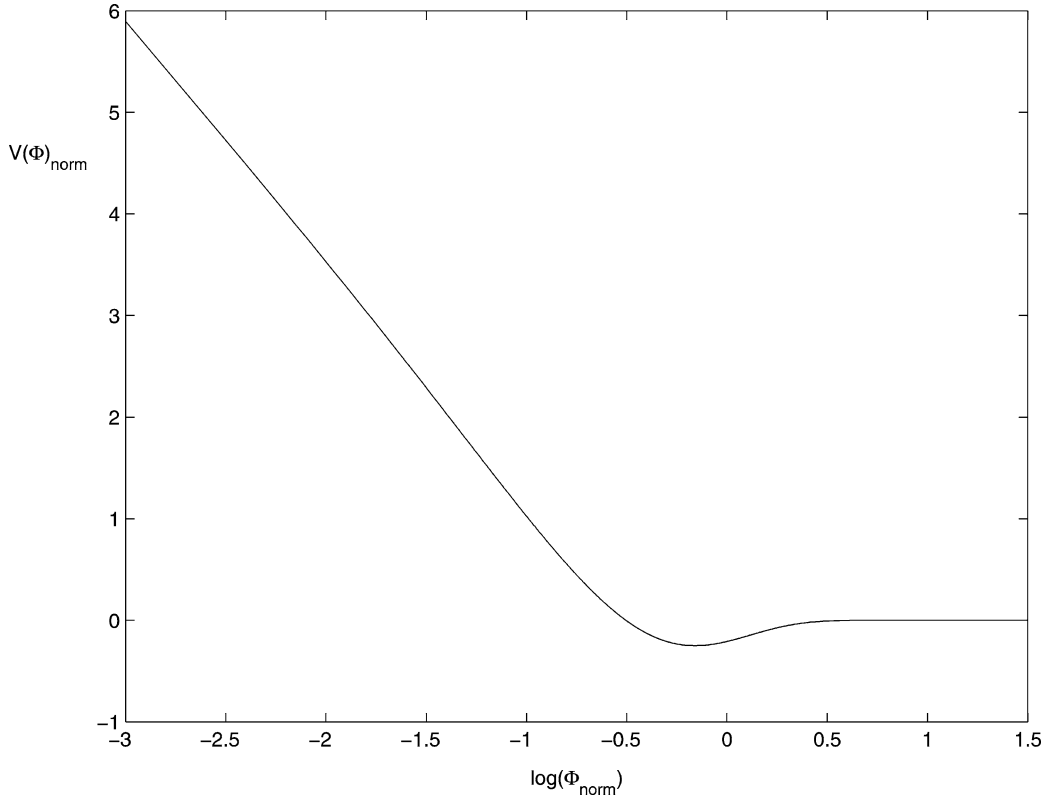


Fig. 1. The scalar potential given by Eq. (19) for $\alpha = \beta$. $\Phi_{\text{norm}} \equiv \sqrt{2\kappa/3}\Phi$ and $V_{\text{norm}} \equiv (2\kappa/\beta)V$.

Arkani-Hamed et al. [23] showed that the cut-off energy is $(m_g^4 M_{Pl})^{1/5}$, where m_g is the mass of the graviton. This is much lower than the Planck scale, and correspondingly, its cut-off length scale is much larger than the Planck length.

In Ref. [19], Nojiri and Odintsov also suggested an extension of the $\ln R$ theory, for which the modified Einstein–Hilbert action reads as

$$L(R) = R - \beta \ln \frac{R}{-\alpha} + \gamma R^m. \quad (22)$$

We would not discuss this model in detail in this Letter. We just note one thing about it. It would correspond to an unique equivalent scalar–tensor formulation if the following equation for ϕ has an unique solution for any value of Φ , see Refs. [13,17]

$$m\gamma\phi^m - \left(\exp\left(\sqrt{\frac{2\kappa}{3}}\Phi\right) - 1 \right)\phi - \beta = 0. \quad (23)$$

Obviously for $m > 1$, this is generally not the case. Thus, generally, the model (22) would not have a well-defined equivalent scalar–tensor theory. What does this imply? According to the analysis of Magnano and Sokolowski [22], this is a strong indication that the original theory is unphysical. Also, for the $R + 1/R + R^2$ theory, for which the same phenomena appears, Flanagan [24] showed that this may imply that the theory has not a well-behaved initial-value formulation. But as indicated by Odintsov [25], this maybe not completely the case. The reason is that it is still unclear which of Einstein or Jordan frame is physical one. For instance, on the classical level the results obtained in these frames (when transformation to equivalent theory exists) are identical even for braneworlds [26]. Of course, on quantum level it is well known (see explicit examples for quantum dilatonic gravity [27–29]) that even classically equivalent theories are not equivalent on quantum level. Hence, the fact that metric theory does not have equivalent classical representation as

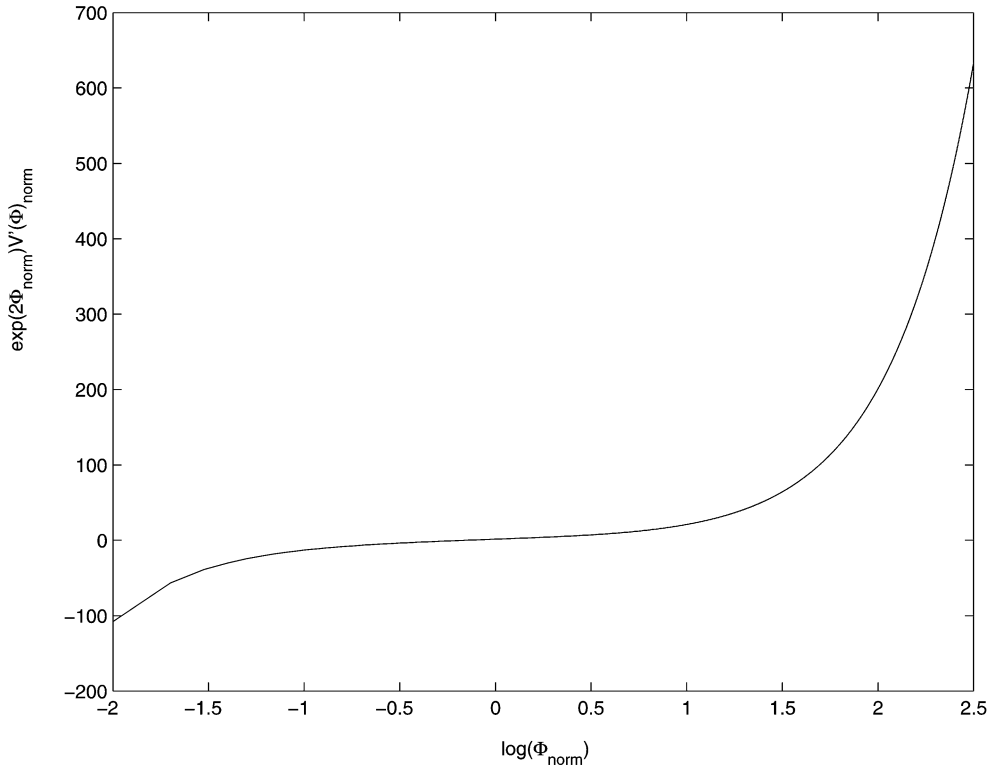


Fig. 2. Derivative of the potential for $\alpha = \beta$, from which combined with the field equation (21) we can determine the evolution of Φ . $\Phi_{\text{norm}} \equiv \sqrt{2\kappa/3}\Phi$ and $V_{\text{norm}} \equiv (2\kappa/\beta)V$.

scalar–tensor theory does not mean that it is ruled out as physical theory.

4. Conclusions and discussions

In this Letter we discussed the Palatini formation of the modified gravity with a $\ln R$ term suggested by Nojiri and Odintsov [19]. We showed that in the Palatini form, the $\ln R$ gravity can drive a current exponential accelerated expansion and it reduces to the standard Friedmann evolution for high redshift region. We discussed the equivalent scalar–tensor formation. We indicated that the $\ln R$ gravity may still have a conflict with electron–electron scattering experiment which stimulates us to pursue a more fundamental theory which can give the $\ln R$ gravity as an effective theory. Finally, we discussed a problem faced with the extension of the $\ln R$ gravity by adding R^m terms. It is clear that many works still need to be done to

see whether the idea of modifying gravity to achieve cosmic acceleration in stead of dark energy is viable.

On gravity theory itself, especially the reasonable form of a quantum gravity is also challenging. With many discussions for extended gravity models [6–8, 16,18,19], we expect the two tales originate from one truth to be discovered.

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